Analysis of Asymmetric GARCH Volatility Models with Applications to Margin Measurement

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Abstract

We explore properties of asymmetric GARCH models in the Threshold GARCH family and propose a more general Spline GTARCH model which captures high frequency return volatility, low frequency macroeconomic volatility as well as an asymmetric response to past negative news in both ARCH and GARCH terms. Based on Maximum Likelihood estimation of S&P 500 returns, S&P/TSX returns and Monte Carlo numerical example, we find that the proposed more general asymmetric volatility model has better fit, higher persistence of negative news, higher degree of risk aversion and significant effects of macroeconomic variables on the low frequency volatility component. We then apply a variety of volatility models in setting initial margin requirements for Central Clearing Counterparties (CCPs). Finally we show how to mitigate procyclicality of initial margins using three regime threshold autoregressive model.

Key Words: CCP Initial Margins, Tail Risk, Risk Aversion, Procyclicality, Threshold GARCH, Spline, Threshold Autoregressive Model.

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1 Introduction

The generalized autoregressive conditional heteroscedasticity (GARCH) model and exponential weighted moving average (EWMA) Riskmetrics model are popular for measuring and forecasting volatility by financial practitioners. Since its introduction there have been many extensions of GARCH models that resulted in better statistical fit and forecasts. For example, GJR-GARCH (Glosten, Jagannathan, & Runkle (1993)) is one of the well-known extensions of GARCH models with an asymmetric term which captures the effect of negative shocks in equity prices on volatility commonly referred to a leverage effect. EGARCH introduced by Nelson (1991) is an alternative asymmetric model of the logarithmic transformation of conditional variance that does not require positivity constraints on parameters. Different volatility regimes can be captured by Markov Regime Switching ARCH and GARCH models allowing for stochastic time variation in parameters. These models were introduced by Cai (1994) and Hamilton and Susmel (1994) correspondingly.

Since tail risk measures typically incorporate forecasts of volatility, model specification is important. Engle and Mezrich (1995) introduced a way to estimate value at risk (VaR) using a GARCH model, while Hull and White (1998) proved that a GARCH model has a better performance than a stochastic volatility model in calculation of VaR. The GJR-GARCH model was also used by Brownlees and Engle (2017) among others for forecasting volatility and measurement of tail and systemic risks.

A typical feature of the GARCH family models is that the long run volatility forecast converges to a constant level. An exception is the Spline-GARCH model of Engle and Rangel (2008) that allows the unconditional variance to change with time as an exponential spline and the high frequency component to be represented by a unit GARCH process. This model may incorporate macroeconomic and financial variables into the slow moving component and as shown in Engle and Rangel (2008) improves long run forecasts of international equity indices. In this model the unconditional volatility coincides with the low-frequency volatility. The Factor-Spline-GARCH model developed in Rangel and Engle (2012) is used to estimate high and low frequency components of equity correlations. Their model is a combination of the asymmetric Spline GJR-GARCH and the DCC (dynamic conditional correlations) models. Another application of an asymmetric Spline GJR-GARCH model for commodity volatilities is in Carpantier and Dufays (2012).

In this paper we generalize the asymmetric Spline-GARCH models using a more general threshold GARCH model as in Goldman (2017). The widely used asymmetric GJR-GARCH model has a problem that the unconstrained estimated coefficient of α often has a negative value for equity indices. A typical solution to this problem is setting the coefficient of α to zero in the constrained Maximum Likelihood or Bayesian estimation. Following Goldman (2017) we use a generalized threshold GARCH (GTARCH) model where both coefficients, α and β, in the GARCH model are allowed to change to reflect the asymmetry of volatility due to negative shocks. We use data for the US and Canadian equity indices, S&P 500 (SPX) and S&P/TSX (TSX), as well as a numerical example to estimate various asymmetric volatility models. We find that the most general GTARCH model fits better as well as does not have a negative alpha bias. We also find higher persistence and more risk aversion in the GTARCH models.

We add macroeconomic variables of gdp growth, inflation, overnight interest rate and exchange rate into the spline model for the slow moving component. The Spline-Macro model results in smaller number of optimal knots for SPX and has better fit for both SPX and TSX.

Next we apply GTARCH, Spline-GTARCH and Spline-Macro-GTARCH models for value at risk (VaR) and conditional value at risk (CVaR) or Expected Shortfall (ES) estimation. For comparison we also estimate Riskmetrics exponential weighted moving average (EWMA), GARCH, GJR-GARCH and GTARCH0 models. In the latter model that we introduce the asymmetric effect of negative news is in the GARCH term but not in the ARCH term. We perform backtest and compare the performance of VaR and ES models using Kupiec (1995) test. We find that all asymmetric volatility models pass Kupiec test for SPX and TSX data while EWMA and GARCH fail the test.

The mandatory use of clearing in certain markets is one of the cornerstone regulations introduced to prevent another global financial crisis. However, the rules implemented have not been tested in crisis conditions. Central Counterparties (CCPs) base their risk management systems on a tiered default waterfall relying on two types of resources provided by their members: margins and default fund contributions. The CCPs, by acting as intermediary, have exposure to both the buyer and the seller. The initial margins are typically set by CCPs based on Value at Risk (VaR) models (Murphy et. al (2016),Knott and Polenghi (2006)).

As documented in Murphy et. al (2014, 2016) and Glasserman and Wu (2017) typical margin models are typically procyclical and may negatively impact members' funding liquidity at the times of crisis. We explore the procyclicality of initial margin requirements based on VaR volatility models above. On the one hand, there is a need for margins to adjust to changes in the market and be responsive to risk. Thus margins are higher in times of stress and lower when volatility is low. However, this practice may produce big changes in margins when markets are stressed which, in turn, may lead to liquidity shocks. In addition, in stable times margins may be too low. CCPs try to reduce the procyclicality of their models by using various methods, including setting floors on margin. Some such methods are discussed in white papers produced by the Bank of England (Murphy et. al (2016)). Their study suggests five tools, including a floor margin buffer of 25% or greater to be used in times of stressed conditions. We suggest placing both a floor and a ceiling on margins, by using a threshold autoregressive model with three regimes (3TAR), as well as expert judgement based on historical margin settings. We estimate 3 TAR for each volatility based VaR model and discuss the resulting regimes and settings for floor and ceiling. If the margins were allowed to be set within two bounds and the high volatility regime was not persistent, margins would be stable. Such policy could be also useful to manage expectations at times of stressed liquidity.

The paper is organized as follows. Section 2 presents GTARCH and Spline GTARCH models, maximum likelihood estimation and tail risks. In Section 3 we perform data analysis for S&P500, S&P/TSX and a numerical example with Monte Carlo simulations. In Section 4 we compare tail risks and perform backtests of all models. Next we analyze procyclicality properties and estimate a three regime TAR model for setting a floor and a ceiling on margins. Section 5 presents conclusion and further work.

2 Asymmetric Threshold GARCH Models

In this section, we present the generalized threshold GARCH model (GTARCH) and a family of its subset models including GJR-GARCH, GTARCH0 and GARCH. Next we add spline to the GTARCH model extedning the analysis of Engle and Rangel (2008).

2.1 The Generalized Threshold GARCH (GTARCH) Model

One of the stylized facts in empirical asset pricing is negative correlation between asset returns and volatility commonly explained by risk aversion and leverage effect. In a popular threshold ARCH or GJR-GARCH model (Glosten, Jagannathan, and Runkle (1993)) a negative return results in an asymmetrically higher effect on the next day conditional variance compared to a positive return.

Consider time series of logarithmic returns r_t with constant mean μ and the GJR-GARCH conditional variance σ_t^2 given by

$$r_t = \mu + u_t = \mu + \sigma_t \varepsilon_t \tag{1}$$
$$\sigma_t^2 = \omega + \alpha \varepsilon_t^2 + \gamma \varepsilon_t^2 I(r_{t-1} - \mu < 0) + \beta \sigma_{t-1}^2$$

where ε_t are Gaussian (or other distribution) independent random variables with mean zero and unit variance, $I(r_{t-1} - \mu < 0)$ is a dummy variable equal to one when previous day innovation u_{t-1} is negative; α and β are GARCH parameters; and γ is an asymmetric term capturing risk aversion. The stationarity condition for the GJR-GARCH model is approximately given by: $1 - \alpha - \beta - \frac{1}{2}\gamma > 0$. [1]

However, there is a problem with the threshold ARCH model above since coefficient α may take negative values in practice. In such case a constrained optimization imposing positivity on all variance parameters results in α equal to zero. Goldman(2017) suggested to use a more general Threshold GARCH or GTARCH model:

$$\sigma_t^2 = \omega + \alpha \varepsilon_t^2 + \gamma \varepsilon_t^2 I(r_{t-1} - \mu < 0) + \beta \sigma_{t-1}^2 + \delta \sigma_{t-1}^2 I(r_{t-1} - \mu < 0), \tag{2}$$

where added term δ reflects degree of asymmetric response in the GARCH term. In this model both parameters γ and δ create the asymmetric response of volatility to negative shocks. Results below show that by allowing both ARCH and GARCH parameters to change with negative news results in better statistical fit and smaller information criteria. Moreover, the GTARCH model not only better captures the leverage effect but also shows higher persistence for negative returns compared to its subset GJR-GARCH model. In addition the coefficients of μ and ω could be allowed to change with regime of negative news to make the model even more flexible. The GTARCH is a

[1] To be more precise the stationarity condition is given by: $1 - \alpha - \beta - \theta\gamma > 0$, where θ is percentage of observations in the regime with negative innovations $u_t < 0$. In practice θ is set to 0.5.

generalized model with the following subset of models: GJR-GARCH ($\delta = 0$), GTARCH0 ($\gamma = 0$) and GARCH ($\gamma = 0$ and $\delta = 0$).

The stationarity condition for the GTARCH model is given by: $1-\alpha-\beta-\frac{1}{2}\gamma-\frac{1}{2}\delta > 0$.[2] The more general GTARCH model due to its flexibility of parameters shows different dynamics for GARCH parameters when the news is negative and allows for higher persistence in the regime of negative news. This in turn takes away the negative bias from α which measures the reaction to the positive news. At the same time estimation of extra parameters using the maximum likelihood is a straightforward extension as shown in Section 2.3.

In addition to GTARCH models we also estimate the Exponentially Weighted Moving Average (EWMA) model defined as:

$$\sigma_t^2 = (1-\lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2; \tag{3}$$

where λ is a smoothing parameter estimated using maximum likelihood. This model is not stable but is a benchmark for 1 day volatility forecasts with typical estimate of $\lambda = 0.94$ frequently used in the industry. The EWMA model is popular for measuring tail risks as will be discussed below.

2.2 The Spline Generalized Threshold GARCH (Spline-GTARCH) Model

The literature incorporating economic variables for modeling and forecasting financial volatility has been growing. For example, Officer(1973), Schwert(1989), Roll (1988), Balduzzi et al. (2001), Anderson et al. (2007) among others found that even though the linkages between aggregate volatility and economy are weak volatility is higher during recessions and post-recessionary stages, and lower during normal periods. Engle and Rangel (2008) introduced the Spline-GARCH model combining high frequency financial returns and low frequency macroeconomic variables.

[2] To be more precise the stationarity condition is $1-\alpha-\beta-\theta\gamma-\theta\delta > 0$

The latter paper analyzes the effects of macroeconomic variables on the slow moving component of volatility using spline. This model releases the assumption of volatility mean reversion to a constant level which is a property of a stable GARCH model. Instead the long-run unconditional variance is dynamic.

The Engle and Rangel (2008) Spline-GARCH model is given by the following returns r_t, GARCH variance σ_t^2 and quadratic spline τ_t equations:

$$\begin{aligned} r_t - E_{t-1} r_t &= \sqrt{\tau_t \sigma_t^2} z_t, \\ \sigma_t^2 &= (1-\alpha-\beta) + \alpha \left(\frac{(r_{t-1} - E_{t-1} r_t)^2}{\tau_{t-1}} \right) + \beta \sigma_{t-1}^2, \\ \tau_t &= c\, exp(w_0 + \sum_{i=1}^{k} w_i ((t - t_{i-1})_+)^2 + m_t \gamma), \end{aligned} \tag{4}$$

$$(t-t_i)_+ = \begin{cases} (t-t_i), & \text{if } t \geq t_i, \\ 0, & \text{otherwise,} \end{cases}$$

where z_t is a standard Gaussian white noise process, σ_t^2 is a GARCH process with unconditional mean of one, m_t is the set of weakly exogenous variables (i.e. macroeconomic variables), and $(t_0 = 0,\ t_1, t_2, ...,\ t_k = T)$ is a partition of total number of observation T into k equal subintervals. The constant term in the GARCH equation is equal to $(1-\alpha-\beta)$ due to the normalization of the GARCH process. Since the constant term in the GARCH variance equation is normalized the long run (unconditional) variance is determined by the spline. Higher number of knots (k) implies more cycles in the low-frequency volatility while parameters $w_1, ..., w_k$ represent the sharpness of the cycles.

We propose the Spline-GTARCH model that accounts for both asymmetric effect in high frequency volatility and the slow moving spline component. Combining the Spline model (4) with the general GTARCH asymmetric volatility model in equation (2) we propose get:

$$
\begin{aligned}
r_t &= \mu + \sqrt{\tau_t \sigma_t^2} z_t, \\
\sigma_t^2 &= \omega + \alpha \left(\frac{(r_{t-1} - \mu)^2}{\tau_{t-1}} \right) + \gamma \left(\frac{(r_{t-1} - \mu)^2}{\tau_{t-1}} \right) I(r_{t-1} - \mu < 0) \\
&\quad + \beta \sigma_{t-1}^2 + \delta \sigma_{t-1}^2 I(r_{t-1} - \mu < 0), \\
\tau_t &= c \, exp(\sum_{i=1}^{k} w_i ((t - t_{i-1})_+)^2 + m_t \gamma),
\end{aligned}
\tag{5}
$$

where $\omega = (1 - \alpha - \beta - \frac{1}{2}\gamma - \frac{1}{2}\delta)$ and $omega > 0$ if the GTARCH process is stable.

In equation (5) we simplified the return process with a constant μ instead of the time variant conditional mean (which could be easily extended for a different process). In practice we also dropped the constant w_0 in the quadratic spline as it was never significant.[3]

The vector of all jointly estimated parameters in the most general model is $\theta = \{\mu, \alpha, \beta, \gamma, \delta, c, w_1, ..., w_k\}$. We note that Spline-GJR-GARCH, Spline-GARCH and Spline-GTARCH0 (the latter has asymmetry only in the GARCH term) are subsets of the model in equation (4) and will be estimated as part of the analysis.

2.3 Maximum Likelihood Estimation

We use the Maximum Likelihood (MLE) to jointly estimate parameters in the Spline-GTARCH model: $\theta = \{\mu, \alpha, \beta, \gamma, \delta, c, w_1, ..., w_k\}$. The positivity and stability restrictions on the parameters are given by $\alpha, \beta, \gamma, \delta \geq 0$ and $\alpha + \beta + 0.5\gamma + 0.5\delta < 1$.

Even though we use a Gaussian process for returns in the likelihood function below the normality assumption is not crucial since asymptotically a quasi-maximum likelihood approach can be used if returns are not Gaussian.

[3] A similar Spline-GARCH specification with constant μ and $w0 = 0$ is used by NYU Stern VLAB Institute at vlab.stern.nyu.edu

The likelihood function is the product of probability density functions:

$$f(r_t;\mu,\sigma_t,\tau_t) = \frac{1}{\sqrt{2\pi\tau_t\sigma_t^2}} e^{-\frac{1}{2\sigma_t^2}\frac{(r_t-\mu)^2}{\tau_t}}.$$

We maximize the log likelihood function below to find estimates of $\hat{\theta}$:

$$L(\hat{\theta}) = log(L(r_t|\theta)) = -\frac{T}{2}log2\pi - \frac{1}{2}\sum_{t=1}^{T}\left(log\sigma_t^2 + log\tau_t + \frac{(r_t-\mu)^2}{\sigma_t^2\tau_t}\right). \qquad (6)$$

The number of knots, k, is chosen by minimizing information criteria: Bayesian-Schwartz information criterion $BIC = -2L(\hat{\theta}) + d \times 2/T$ and Akaike information criterion $AIC = -2L(\hat{\theta}) + d \times ln(T)/T$, where d is the dimension of $\hat{\theta}$. In addition to selecting number of knots in the spline we use the above criteria for comparing overall fit of various volatility models discussed in this paper. We note that as penalty on the degrees of freedom is higher for the BIC using this criterion can result in choosing a more parsimonious model with smaller number of parameters.

2.4 Tail Risks

One of the most popular tail risk measures is $q\%$ value at risk (VaR) which is defined as a loss of the portfolio that can be exceeded only with probability $1-q$. For a unit value of the portfolio essentially VaR is the negative of $(1-q)$ quantile of the distribution of returns, where q is the upper tail probability.[4]

$$P\left(r_t < -VaR_q\right) = 1-q.$$

Both in-sample and out-of-sample daily VaR can be computed based on volatility model used for estimation and forecasting of portfolio returns. The VaR is typically computed using either a parametric assumption for the distribution of returns or bootstrapped standardized residuals (also called filtered historical simulation).

[4]Since VaR is reported as a positive number it is typically measured as negative of a 1%, 5% or 10% quantile.

If a parametric assumption is used with a cumulative density function F the 1 day $q\%$ VaR is given by:

$$VaR_{t+1} = \sigma_{t+1} \times F^{-1}_{(1-q)},$$

where $F^{-1}_{(1-q)}$ is the $(1-q)$ quantile of the distribution of F. If a standard normal distribution is used for F the daily VaR can be estimated based on $F^{-1}_{(1-q)} = 1.282, 1.645, 2.326$ for $q = 90\%, 95\%, 99\%$ respectively.

If the standardized residuals $e_t = \frac{r_t - \mu}{\sigma_t}$ after adjustment for time-varying volatility still have fat tails the alternative approach is to use bootstrap or filtered historical simulation (FHS) based on Hull and White (1998) paper. They suggest to estimate the daily VaR through a filtered process by estimating the F's quantile instead of using parametric distributional assumption. The estimate of F^{-1}_{1-q} is the $1-q$ quantile of the empirical distribution of the standardized residuals e_t.

In an extreme outcome of $1-q$ probability the actual loss (L) is larger than VaR, especially, when the loss distribution has a very long tail. An alternative commonly used tail risk is conditional VaR (CVaR) or expected shortfall (ES) which measures the expected value of the portfolio loss given the loss actually exceeded the VaR.

The ES is given by

$$ES_{1-q} = E(L|L > VaR_{1-q}).$$

Similar to VaR we can apply a parametric or Hull and White (1998, HW) method to estimate the expected shortfall. In case of normal distribution it is given by

$$ES_{1-q} = \frac{\phi(VaR_{1-q})}{q} \times \sigma_t,$$

where ϕ is standard normal probability density function.

For the HW method we sort the standardized residuals and find the average of them in the $1-q$ percent tail. Then we multiply this value by the 1 day forecast of volatility.

3 Data Analysis

In this section we perform data analysis for S&P500 (SPX), S&P/TSX (TSX) and a numerical example with Monte Carlo simulations. The results of thirteen estimated volatility models are discussed below. The daily SPX data for the period between 10/08/2002-12/30/2016 was obtained from CRSP in Wharton Database, while the TSX data for the period between 03/17/2003-03/31/2017 was obtained from Bloomberg. For both series we found logarithmic returns that resulted in 3500 observations.

For the spline model with macroeconomic variables we used similar data to Engle and Rangel (2008) including quarterly nominal GDP growth rates for both countries; daily US federal funds effective rate and Canadian overnight money market financing rate; monthly CPI inflation for both countries; daily Trade Weighted U.S. Dollar Index and USD/CAD exchange rate. We also added monthly unemployment rate for each country. Table 1 provides the description and sources of data for all variables.

Table 1 here

We transformed macroeconomic variables for the Spline-Macro model in the following way. For the CPI, GDP and exchange rates we used log differences, while interest rates and unemployment rates were used at levels. We ran AR(1) model for each variable and found the squared residuals. Using the squared residuals we computed the moving average volatility for each variable. For quarterly GDP data, monthly CPI and monthly unemployment data we used 250 days moving average window, while for daily data we use 25 days window.

Tables 2 in Panel A presents the results of estimated simple GTARCH family models without spline for SPX data. We estimate GTARCH with all parameters $(\alpha, \beta, \gamma, \delta)$; GJR-GARCH $(\delta = 0)$, GTARCH0 $(\gamma = 0)$ and GARCH $(\gamma = 0$ and $\delta = 0)$. First we performed unconstrained optimization without imposing a positivity constraint on parameters and then we constrained all parameters to be positive. For the unconstrained results we see that $\alpha = -0.0139$ and is statistically significant in the GJR-GARH model. Clearly GJR-GARCH model does not effectively capture the risk aversion in a single asymmetric parameter γ shifting α to a negative value in order to distinguish better negative and positive news. However, the interpretation of negative α that positive news reduces volatility next period is unintuitive. At the same time α is positive and not significant in the more general GTARCH model. Since GARCH parameters need to be positive we impose constraints in optimization which results in estimated α being positive but very close to zero in these models. Most model parameters are not affected by imposing the positivity constraint except for the GJR-GARCH model.

In the GTARCH model both coefficients $\gamma = .13$ and $\delta = .16$ are highly significant showing the asymmetric effect present in both ARCH and GARCH terms and showing higher persistence in the regime of negative news. Based on both AIC and SIC criteria a GTARCH model is chosen among alternatives. All models satisfy stationarity condition with overall $persistence = \alpha + \beta + \frac{1}{2}\gamma + \frac{1}{2}\delta < 1$.

In order to check the robustness of MLE algorithm we performed Monte Carlo experiments and present an example in Panel B of Table 1. We used previously estimated GTARCH parameters for the SPX as true parameters of the data generating process in this example. We generated data using equation (2) with 5000 observations and 500 replications. For each replication of the date we estimated each model in the GTARCH family and presented means and standard errors of the overall results. For the unconstrained optimization we still find a negative α for the GJR-GARCH model while the GTARCH model has a small positive α. The parameters of constrained GTARCH model are very close to true values (within one standard deviation). The parameters of subset models (GJR-GARCH, GTARCH0 and GARCH) produce biases due to some dropped parameters in their specification.

Table 2 here

Tables 3 and 4 show results of estimated GTARCH family models including specifications without spline, with spline and with spline and macroeconomic variables in equation (5). Table 3 is for SPX while Table 4 is for TSX. Only results with imposed positivity constraints are reported.

In addition to volatility models presented in the table we estimated the Riskmetrics EWMA volatility model that is commonly used as a benchmark in volatility forecasting and value at risk estimation. The MLE for the EWMA model resulted in the following smoothing parameters with standard error given in brackets and information criteria:

SPX: $\lambda = 0.9409$ $(0.0049), AIC = 2.7262, BIC = 2.7279$

TSX: $\lambda = 0.9369$ $(0.0055), AIC = 2.8222, BIC = 2.8240$

The smoothing parameter is very close to .94 in both cases, which is frequently used in practice. Thus the US and Canadian indices have similar EWMA volatility dynamics.

Based on the BIC information criterion with heavier penalty for extra parameters the GTARCH model without spline is preferred, while using the AIC criterion the Spline-Macro-GTARCH is the superior model. This result holds for both SPX and TSX. Note that both selected models include the most general GTARCH specification with presence of asymmetry in both ARCH and GARCH terms. Moreover, the asymmetric term δ goes up to 0.24 in the SPX Spline GTARCH model making the response to negative news even more asymmetric compared to GTARCH without spline with $\delta = 0.16$. The optimal number of knots in the SPX Spline model is 17, while the number of knots goes down to 8 when we add macroeconomic variables. Macroeconomic variables are useful in modeling the low frequency component as their presence reduces the number of knots for cycles and they have statistically significant effect on long run volatility dynamics. The following variables are statistically significant with 10% for predicting the low frequency volatility component for SPX:

- Interest rate ($InterestR$) and interest rate volatility ($InterestR_V$). Both have positive effect on SPX volatility

- Volatility of the unemployment rate ($unempv_V$) has a negative effect on SPX volatility

- Volatility of USD trade weighted index (USD_V) has a positive effect on SPX volatility

- GDP growth has a negative effect on SPX volatility

All the signs are as expected except for the volatility of the unemployment rate. It might be the case that the reduction in unemployment rate rather than increase in it is driving this results.

As for the Canadian data fewer macroeconomic variables were found significant at 10% and the optimal number of knots stays the same (15 knots) after macroeconomic variables are added.

- Inflation Volatility ($Inflation_V$) has a negative effect on TSX volatility

- Interest rate volatility ($InterestR_V$) has a negative effect on TSX volatility

- Volatility of USD/CAD exchange rate ($USDCAD_V$) has a positive effect on TSX volatility

While the effect of Canadian dollar exchange rate volatility has expected positive sign, the negative signs for volatilities of inflation and interest rate are not intuitive. Canadian overnight interest rate was volatile before the crisis in our sample between 2003-2007. During the 2008-2009 crisis and afterwards interest rate was stable. Thus, during the time of high equity volatility interest rates were not volatile compared to previous period. Similarly while the US experienced the largest volatility of inflation (from inflation to deflation) during the 2008-2009 financial crisis, the largest drops in consumer prices in Canada happened in Quarter 1 2008 (before the crisis) and in Quarter 3 2012 which were relatively calm periods in the equity market.

Table 5 shows the degree of risk aversion in each model measured by the correlation between returns r_{t-1} and log difference of fitted conditional variance $log(\sigma_t^2/\sigma_{t-1}^2)$ for each model. The more negative correlation implies higher degree of risk aversion because of asymmetrically higher volatility for negative returns. For comparison the log difference in VIX index has correlation with the S&P 500 return of about -0.7. Table 5 shows that the highest degree of risk aversion is captured by the GTARCH models and the smallest correlation is for the EWMA and GARCH models that are symmetrical.

Tables 3, 4, and 5 here

Figures 1, 2, 3, and 4 here

Figures 1 and 2 show annualized GTARCH volatilities for SPX data while Figures 3 and 4 present similar graphs for TSX. First we present the Spline-GTARCH volatility compared to a simple GTARCH in Figures 1 and 3. Next we compare Spline-Macro-GTARCH volatility to a simple GTARCH in Figures 2 and 4. We can see that the low frequency component is smooth for both SPX and TSX data in the Spline-GTARCH model and the high frequency component is close but generally higher than GTARCH. Once the macroeconomic variables are added the dynamics of both low and high frequency volatilities becomes less smooth with higher peaks. This is due to reaction to macroeconomic volatility in turbulent times affecting both the long run volatility and GTARCH component. The reaction to the negative news is also amplified by the asymmetric effect in the GTARCH model.

Overall the US and Canadian markets volatilities have similar dynamics and peaks, however, the Canadian market has lower level of volatility. For the low frequency spline component the highest level during the financial crisis was 17% for TSX compared to over 40% for SPX. Similarly the high frequency TGARCH volatility peaks in the US market are more than twice of the Canadian market.

4 Initial Margin Measures

In this section we compute tail risks and perform backtests of all models. Next we analyze initial margin models procyclicality and estimate a three regime threshold autoregressive model (3TAR) for setting a floor and a ceiling on margins.

4.1 Properties of Tail Risks for Setting Margin Requirements

Figures 5 and 6 show the logarithmic returns in red and negative values of 1 day 99% Value at Risk (VaR) for SPX and TSX correspondingly. We generated 1 day 99% VaRs using Hull and White (1998) bootstrap method (the blue line) and Normal Distribution (the red line). We used the Spline-GTARCH model on this graphs while all other models are reported in Tables 6 and 7. The margin requirements with Hull and White method are higher because this method uses the actual returns distribution with fat tails compared to Normal distribution.

Tables 6 and 7 here

Figures 5 and 6 here

Table 6 presents 1 to 3 day forecasts of all volatility models, VaR and ES produced by each model for SPX and TSX at the time of low volatility at arbitrarily selected dates in 2016 and 2017. Table 7 reports the same results at the time of high volatility in the Fall of 2008. Margins are usually computed over some period of time greater than one day. For example, exchange traded assets are cleared within 2-3 days in the US. Thus in Tables 6 and 7 we presented 1 to 3 days tail risks that can be easily extended to longer periods. One day VaR and ES at $q = (90\%, 95\%, 99\%)$ are reported using Hull and White (1998) method.[5] In order to compute t-day VaR and ES we used $\sqrt{t}$ adjustment based on Basel requirement and common practice.[6] Monte Carlo simulations would be an interesting extension of the method especially for longer time horizons.

The results for the SPX and TSX data in Table 6 show increasing volatility forecasts from 1 to 3 days since the starting point is at the time of low volatility and volatility is mean-reverting.[7] Similarly volatility forecasts go down in Table 7 when we start in high volatility period. While there is no specific one model which always has the highest volatility forecast and tail risks among reported models the ones with asymmetric terms (GTARCH, GJR-GARCH and GTARCH0) produce higher forecasts and tail risks than symmetrical GARCH and EWMA models. Thus models accounting for risk aversion such as GTARCH are useful to make sure that volatility is well measured and sufficient margin requirements are set.

4.2 Backtesting

Backtesting is often used in practice for model validation. The testing window is set to evaluate number of Value at Risk violations (or breaches) and compare it to expected number of violations for specific VaR quantile. For example, if VaR is measured with $q = 99\%$ the expected number of violations is 1%. Considering the whole sample size of N=3500 observations we would expect 35

[5] To save space we did not report the results with normal distribution which have similar pattern but lower estimates as shown in Figures 6 and 7.

[6] The $\sqrt{t}$ multiplier is correct only under the assumption of independence in returns.

[7] This is true for all models except for EWMA which is not stationary and thus only 1 day volatility forecast can be produced.

violations.[8] If the actual breach rate turns out to be too high the VaR margin model underestimates risk which creates a loss for the CCP. Alternatively, if the breach rate is too low the VaR model overestimates risk and results in unnecessary high margin charges for the members of the CCP. Thus margins can be set based based on VaR that has reasonable number of backtest violations falling within some confidence interval.

The most popular backtesting statistical test used in practice is Kupiec (1995) proportion of failures (POF) test with the null hypothesis that the breach rate is equal to expected $(1-q)\%$ quantile. The two-sided test has asymptotic likelihood ratio statistics with chi-square distribution and one degree of freedom $X^2(1)$.

Table 8 presents the results of backtesting with number of breaches for the 90%, 95% and 99% VaRs of SPX and TSX produced by each volatility model for the whole sample period. The table also shows 95% Confidence Intervals with lower and upper bounds for the number of allowed breaches using Kupiec test. We report the results for VaRs using Hull and White (1998) filtered historical simulations method to make sure that risk is not underestimated compared to models that use Normal distribution.

Table 8 here

The results in Table 8 show that all VaRs that use asymmetric volatility models (GTARCH, GJR-GARCH and GTARCH0) with various quantiles ($q = 90\%, 95\%, 99\%$) pass the Kupiec test at 5% significance level for both SPX and TSX. At the same time we find that the EWMA model failed the test underestimating risk for both SPX and TSX for each quantile q. The GARCH model fails the Kupiec test only for SPX data with $q = 90\%$ quantile overestimating risk.

[8] A common criticism of backtesting is small expected number of violations if the testing window is not large enough.

The backtesting results reenforce the need to use asymmetric volatility models capturing risk aversion to make sure that margins are set adequately.

4.3 Procyclicality of the CCP's Initial Margin Requirements

Central Counterparties (CCPs) base their risk management systems on a tiered default waterfall relying on two types of resources provided by their members: margins and default fund contributions. The initial margins are typically set based on Value at Risk (VaR) calculations. The CCPs, by acting as intermediary, have exposure to both the buyer and the seller. Since VaR and ES calculations are typically volatility based the properties of the underlying volatility models such as risk aversion are essential for setting initial margin requirements.

This section explores the procyclicality of margin requirements based on VaR models and suggests remedies to reduce procyclicality. On the one hand, there is a need for margins to adjust to changes in the market and be responsive to risk. Thus margins are higher in times of stress and lower when volatility is low. However, this practice may produce big changes in margins when markets are stressed which, in turn, may lead to liquidity shocks. In addition, in stable times margins may be too low. CCPs try to reduce the procyclicality of their models by using various methods, including setting floors on margin. Some such methods are discussed in white papers produced by the Bank of England (Murphy et. al (2016)). Their study suggests five tools, including a floor margin buffer of 25% or greater to be used in times of stressed conditions.

We suggest placing both a floor and a ceiling on margins, by using a threshold autoregressive model with three regimes, as well as expert judgement based on historical margin settings. For example, we can evaluate the appropriateness of the suggested 25% margin buffer for maintaining funding liquidity under stressed market. We illustrate the use of this method below.

4.3.1 Threshold Autoregressive Model (TAR)

The Threshold Autoregressive Models (TAR) or Self-Exciting Threshold Models (SETAR) were first introduced by Tong (1977) and Tong and Lim (1980). A smooth transition model (STAR) was later developed by Terasvirta (1994).

Consider a time series of logarithm of Value at Risk, $y_t = log(VaR)$, with three regimes. A simple threshold autoregressive model (TAR) with p lags for y_t is given by:

$$\begin{aligned} y_t &= \phi_0^j + \phi_1^j y_{t-1} + \ldots + \phi_p^j y_{t-p} + \varepsilon_t \qquad (7) \\ \varepsilon_t &\sim \mathrm{N}\left(0, \sigma^2\right), \end{aligned}$$

where $j = 1, \ldots K$ with number of regimes $K = 3$. The regimes are determined by an observable threshold variable z_{t-d} with delay parameter d and sorted threshold values $\theta_1, \ldots, \theta_{K-1}$, such that

$$\begin{cases} j = 1 & z_{t-d} < \theta_1, \\ j = 2 & \theta_1 \leq z_{t-d} \leq \theta_2, \\ j = 3 & z_{t-d} > \theta_2. \end{cases}$$

In practice we use $z_{t-d} = y_{t-d}$ and we set the delay parameter for the threshold variable equal to one ($d = 1$). We also use $p = 2$ for the order of the autoregressive model. Alternatively, these parameters as well as number of regimes K could be found by minimizing information criteria.

While there are various methods[9] to estimate this model the commonly used classical method is a grid search for optimal thresholds $\theta_1, \ldots, \theta_{K-1}$ by minimizing the sum of squared residuals.

We estimated a Threshold Autoregressive Model with three regimes and two corresponding thresholds for the logarithm of VaR for each model. VaR was previously estimated using Hull and White (1998) method. We used log(VaR) for estimation of the 3TAR model since log transformation smoothes the peaks. Then we exponentially transformed the threshold values and reported

[9] For example, Goldman et al (2013) introduced a Bayesian method for measuring thresholds and long memory parameters in a more sophisticated threshold model.

them in Table 9.

The results for thresholds for all volatility models are given in Table 9 and results are presented graphically as horizontal lines in Figures 7 and 8 for log(VaR) in the Spline-GTARCH model for SPX and TSX respectively.

Table 9 here

Figures 7 and 8 here

The three-regime threshold model provides a straightforward method of setting both the floor and the ceiling for the initial margin that is stable and not too procyclical: the one day margins are on average bounded between 1.84% and 2.58% for SPX and between 0.77% and 1.01% for TSX. This way when volatility is low the margins are fixed at a conservative floor level that corresponds historically to about 29% quantile of lowest margins for SPX and at the time of market stress they can't go above the upper threshold. It is an interesting coincidence that the estimated lower threshold for SPX using EWMA model corresponds to the 25% of observations in the lower regime as was also suggested by Murphy et. al (2016). For TSX the margin buffer is a higher 32% of observations on average. On the other hand, at the time of stress the higher regime thresholds on average correspond to 38% of the observation points for both SPX and TSX, which may not appear too conservative. One could add here actual historical margins set by CCPs at the time of stress, to see if the upper bound was historically higher and would have resulted in a lower percentage of observations for the high regime.

In order to guarantee that the margin floors and ceilings would be sufficient at the time of crisis, we need to make sure that the time series of VaRs in the regime of high volatility are stationary and revert back inside the bounds. The unit root test results indicate that all models pass the stability test for the SPX, while EWMA and all spline models without macroeconomic variables could be not reliable to set a sustainable ceiling.

If the margins were allowed to be set within two bounds and the high volatility regime was not persistent, margins would be stable. Such policy could be also useful to manage expectations at times of stressed liquidity.

The mandatory use of CCP's in certain markets is one of the cornerstone regulations introduced to prevent another global financial crisis. However, the rules implemented have not been tested in crisis conditions. Above we presented a simple approach to test the sustainability of margin models using a three regime threshold autoregressive model.

5 Conclusion and Further Development

In this paper we considered asymmetric GARCH models in the Threshold GARCH family and propose a more general Spline GTARCH model which captures high frequency return volatility, low frequency macroeconomic volatility as well as an asymmetric response to past negative news in both ARCH and GARCH terms.

Based on Maximum Likelihood estimation of S&P 500 returns, S&P/TSX returns and Monte Carlo numerical example, we found that the proposed more general asymmetric volatility model has better fit, higher persistence of negative news, higher degree of risk aversion and significant effects of macroeconomic variables on the low frequency volatility component.

We then apply a variety of volatility models including asymmetric GARCH, GARCH and EWMA in setting initial margin requirements for Central Clearing Counterparties (CCPs). Since VaR and ES calculations are typically volatility based the properties of the underlying volatility models such as risk aversion are essential for setting initial margin requirements.

Finally we show how to mitigate procyclicality of initial margins using three regime threshold autoregressive model.

In the future research more international equity markets can be tested and additional macroeconomic variables can be added to the spline. The VaR bootstrap algorithm can be modified to the one with rolling windows. The multi-day VaR and ES multiplier could be computed using Monte Carlo simulations. Additional back tests including Christofferson (2004) could be applied to test that breaches are not clustered. In terms of margin pro-cyclicality mitigation a Markov-Switching model with three regimes can be applied as well and compared to the threshold autoregressive model.

Table 1: **Definitions of Variables and Data Sources**

Definition	Frequency	Source
The US Data		
S&P500 composite index	Daily (business)	CRSP Wharton Database
US federal funds effective rate	Daily (business)	FEDL01 Index (Bloomberg)
US nominal GDP	Quarterly	U.S. Department of Commerce, BEA.
US CPI, chained	Monthly	U.S. Bureau of Labor Statistics
Unemployment rate	Monthly	U.S. Bureau of Labor Statistics
Trade Weighted U.S. Dollar Index: Major Currencies	Daily	DTWEXM St Louis FED
The Canadian Data		
S&P/TSX composite index	Daily (business)	SPTSX Index (Bloomberg)
Canadian overnight money market financing rate	Daily (business)	CAOMRATE Index (Bloomberg)
Canadian nominal GDP	Quarterly	CANSIM table 380-0064.
Canadian CPI	Monthly	CANSIM table 326-0022.
Unemployment rate	Monthly	CANSIM table 282-0087.
Units of USD per CAD	Daily(business)	CAD-USAD X-RATE-Price (Bloomberg)

Table 2: **Estimation Results for GTARCH Models: SPX and Monte Carlo Example**

		GTARCH				GTARCH0				GJR-GARCH				GARCH			
	True	unconstrained		constrained		unconstrained		constrained		unconstrained		constrained		unconstrained		constrained	
	parm	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std
Panel A: SPX Results																	
μ		0.0076	(0.0126)	0.0003	(0.0000)	0.0256	(0.0126)	0.0006	(0.0000)	0.0164	(0.0137)	0.0184	(0.0134)	0.0546	(0.0130)	0.0546	(0.0140)
ω		0.0218	(0.0029)	0.0226	(0.0037)	0.0220	(0.0030)	0.0226	(0.0031)	0.0227	(0.0032)	0.0238	(0.0039)	0.0238	(0.0039)	0.0238	(0.0040)
α		0.0007	(0.0081)	0.0000	(0.0130)	0.0833	(0.0085)	0.0780	(0.0082)	-0.0139	(0.0069)	0.0000	(0.0194)	0.1015	(0.0110)	0.1015	(0.0110)
β		0.8357	(0.0153)	0.8374	(0.0187)	0.7823	(0.0150)	0.7887	(0.0153)	0.8978	(0.0114)	0.8879	(0.0187)	0.8755	(0.0123)	0.8755	(0.0125)
γ		0.1370	(0.0173)	0.1398	(0.0197)					0.1849	(0.0189)	0.1745	(0.0213)				
δ		0.1634	(0.0240)	0.1596	(0.0248)	0.2460	(0.0237)	0.2485	(0.0246)								
Persistence		0.9866		0.9871		0.9886		0.9909		0.9763		0.9752		0.9770		0.9770	
BIC		2.3232		2.3239		2.3444		2.3442		2.3344		2.3353		2.3714		2.3714	
AIC		2.3126		2.3133		2.3356		2.3354		2.3256		2.3265		2.3644		2.3644	
Panel B: Monte Carlo Simulations																	
μ	0.0076	0.0030	(0.0395)	0.0100	(0.0111)	0.0196	(0.0156)	0.0205	(0.0135)	0.0145	(0.0150)	0.0164	(0.0129)	0.0594	(0.0129)	0.0592	(0.0131)
ω	0.0218	0.0275	(0.0516)	0.0222	(0.0027)	0.0244	(0.0229)	0.0221	(0.0036)	0.0231	(0.0037)	0.0231	(0.0036)	0.0262	(0.0057)	0.0259	(0.0062)
α	0.0007	0.0026	(0.0227)	0.0039	(0.0058)	0.0725	(0.0090)	0.0728	(0.0098)	-0.0095	(0.0046)	0.0006	(0.0022)	0.1282	(0.0139)	0.1267	(0.0188)
β	0.8357	0.8290	(0.0622)	0.8313	(0.0121)	0.7691	(0.0627)	0.7749	(0.0155)	0.8911	(0.0183)	0.8841	(0.0177)	0.8539	(0.0141)	0.8555	(0.0202)
γ	0.1370	0.1375	(0.0177)	0.1342	(0.0138)					0.0145	(0.0104)	0.0164	(0.0086)				
δ	0.1634	0.1612	(0.0392)	0.1676	(0.0224)	0.2669	(0.0454)	0.2710	(0.0240)								
Persistence	0.9866	0.9809		0.9860		0.9751		0.9832		0.8888		0.8929		0.9821		0.9822	
BIC		2.3869		2.3812		2.4094		2.3997		2.3932		2.3939		2.4394		2.4418	
AIC		2.3763		2.3707		2.4006		2.3909		2.3844		2.3851		2.4323		2.4347	

Notes: Panel A presents the results of estimation of GTARCH models for SPX data between 10/08/2002-12/30/2016 with 3500 observations. Panel B presents results of Monte Carlo simuations using parameters of estimated SPX model for data generating process. We used sample size of N=5000 and 500 replications.

Table 3: **Estimation Results for GTARCH, Spline-GTARCH and Spline-Macro-GTARCH Models: SPX**

Parm	GTARCH						GTARCH0						GJR-GARCH						GARCH					
	No Spline		Spline		SMacro		No Spline		Spline		SMacro		No Spline		Spline		SMacro		No Spline		Spline		SMacro	
	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std
μ	0.000	(0.000)	0.008	(0.015)	0.013	(0.015)	0.001	(0.000)	0.026	(0.014)	-0.726	(0.154)	0.018	(0.013)	0.025	(0.012)	0.028	(0.013)	0.055	(0.014)	0.058	(0.013)	0.058	(0.013)
ω	0.023	(0.004)					0.022	(0.003)					0.023	(0.003)					0.023	(0.003)				
α	0.000	(0.013)	0.000	(0.022)	0.000	(0.023)	0.078	(0.008)	0.065	(0.006)	0.061	(0.018)	0.000	(0.019)	0.000	(0.000)	0.000	(0.000)	0.102	(0.011)	0.097	(0.009)	0.089	(0.010)
β	0.837	(0.019)	0.771	(0.028)	0.781	(0.030)	0.789	(0.015)	0.734	(0.015)	0.729	(0.033)	0.888	(0.019)	0.840	(0.013)	0.841	(0.016)	0.875	(0.012)	0.833	(0.015)	0.826	(0.018)
γ	0.140	(0.020)	0.131	(0.025)	0.122	(0.023)							0.175	(0.021)	0.189	(0.017)	0.177	(0.020)						
δ	0.160	(0.025)	0.243	(0.033)	0.230	(0.035)	0.249	(0.025)	0.310	(0.025)	0.316	(0.031)												
c			2.718	(0.222)	0.936	(0.289)			1.800	(0.300)	0.846	(0.513)			2.246	(0.418)	0.754	(0.193)			1.873	(0.373)	0.640	(0.158)
w_1			-1.973	(0.236)	-0.711	(0.118)			-1.454	(0.370)	-0.726	(0.154)			-2.215	(0.426)	-0.760	(0.106)			-1.994	(0.462)	-0.753	(0.107)
w_2			4.110	(0.626)	0.854	(0.251)			2.860	(1.030)	0.821	(0.448)			4.587	(1.211)	1.032	(0.241)			4.196	(1.281)	0.891	(0.222)
w_3			-2.430	(0.712)	2.106	(0.567)			-1.300	(1.157)	2.365	(0.838)			-2.706	(1.591)	1.854	(0.618)			-2.589	(1.516)	2.301	(0.566)
w_4			-0.008	(0.700)	-4.443	(0.743)			-0.894	(0.900)	-4.796	(0.936)			0.185	(1.765)	-4.267	(0.804)			0.099	(1.424)	-4.759	(0.759)
w_5			1.669	(0.679)	2.831	(0.455)			2.948	(0.872)	2.918	(0.468)			1.645	(1.704)	2.752	(0.486)			2.109	(1.436)	2.917	(0.464)
w_6			-1.607	(0.687)	-1.002	(0.398)			-3.167	(1.101)	-0.872	(0.569)			-2.343	(1.519)	-0.972	(0.380)			-2.689	(1.521)	-0.964	(0.353)
w_7			1.628	(1.152)	0.736	(0.527)			3.230	(1.157)	0.651	(0.791)			3.429	(1.782)	0.837	(0.405)			3.071	(1.652)	0.911	(0.365)
w_8			-5.859	(1.744)	-0.859	(0.728)			-7.984	(1.153)	-1.153	(1.161)			-9.085	(2.101)	-1.130	(0.559)			-8.526	(2.104)	-1.532	(0.506)
w_9			6.568	(1.585)					8.707	(0.996)					10.785	(1.950)					10.411	(2.108)		
w_{10}			-1.421	(0.907)					-2.060	(0.763)					-5.566	(1.875)					-4.571	(1.650)		
w_{11}			-1.728	(0.815)					-2.658	(0.803)					2.000	(2.214)					0.363	(1.501)		
w_{12}			1.752	(0.815)					2.842	(0.783)					-1.081	(2.132)					-0.282	(1.476)		
w_{13}			-1.128	(0.782)					-1.278	(1.194)					0.239	(1.876)					0.904	(1.412)		
w_{14}			0.175	(0.796)					-0.011	(1.153)					-0.013	(1.834)					-0.960	(1.667)		
w_{15}			2.089	(1.145)					1.689	(1.088)					1.971	(2.014)					2.481	(1.911)		
w_{16}			-3.193	(1.194)					-3.076	(1.212)					-3.218	(2.222)					-4.310	(1.951)		
w_{17}			0.005	(1.105)					0.523	(1.220)					-0.201	(2.488)					1.848	(2.378)		
$Inflation$					0.084	(0.109)					0.081	(0.150)					0.112	(0.103)					0.062	(0.092)
$Inflation_V$					-1.309	(0.829)					-1.983	(0.748)					-1.190	(1.150)					-1.983	(1.122)
$InterestR$					0.675	(0.152)					0.749	(0.165)					0.666	(0.152)					0.768	(0.149)
$InterestR_V$					2.077	(1.166)					2.050	(2.079)					4.326	(2.033)					4.972	(1.541)
$unemp_V$					-1.456	(0.779)					-5.099	(1.911)					-5.132	(4.081)					-6.893	(3.957)
USD_V					0.992	(0.332)					1.176	(0.329)					1.028	(0.315)					1.167	(0.305)
GDP					-0.218	(0.088)					-0.250	(0.154)					-0.245	(0.087)					-0.269	(0.076)
GDP_V					0.224	(0.216)					0.500	(0.247)					0.403	(0.235)					0.601	(0.231)
$Persistence$	0.987		0.958		0.957		0.991		0.953		0.948		0.975		0.935		0.929		0.977		0.930		0.915	
BIC	2.324		2.333		2.330		2.344		2.357		2.350		2.335		2.346		2.343		2.371		2.390		2.382	
AIC	2.313		2.292		2.291		2.336		2.319		2.313		2.335		2.309		2.308		2.326		2.353		2.347	

Notes: This table presents the results of all volatility models for SPX. SPX data are for the period between 10/08/2002-12/30/2016. The sample size is 3500 observations. Only results with positivity constraints are reported. In addition to volatility models presented in the table we estimated EWMA model that resulted in smoothing parameter estimate and standard error given in brackets: $\lambda = 0.9409$ (0.0049) and information criteria: $AIC = 2.7262, BIC = 2.7279$.

Table 4: **Estimation Results for GTARCH, Spline-GTARCH and Spline-Macro-GTARCH Models: TSX**

Parm	GTARCH						GTARCH0						GJR-GARCH						GARCH					
	No Spline		Spline		SMacro		No Spline		Spline		SMacro		No Spline		Spline		SMacro		No Spline		Spline		SMacro	
	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std
μ	0.000	(0.000)	0.010	(0.005)	0.013	(0.006)	0.000	(0.000)	0.016	(0.005)	-1.256	(0.772)	0.014	(0.005)	0.015	(0.005)	0.016	(0.005)	0.024	(0.005)	0.025	(0.005)	0.026	(0.005)
ω	0.002	(0.000)					0.002	(0.000)					0.003	(0.001)					0.002	(0.001)				
α	0.019	(0.009)	0.000	(0.011)	0.000	(0.062)	0.058	(0.007)	0.046	(0.005)	0.045	(0.010)	0.010	(0.009)	0.000	(0.000)	0.000	(0.000)	0.085	(0.010)	0.099	(0.010)	0.077	(0.008)
β	0.869	(0.013)	0.841	(0.014)	0.826	(0.058)	0.844	(0.013)	0.808	(0.020)	0.796	(0.023)	0.914	(0.011)	0.872	(0.013)	0.861	(0.015)	0.902	(0.012)	0.890	(0.011)	0.838	(0.017)
γ	0.069	(0.015)	0.076	(0.016)	0.079	(0.047)							0.107	(0.015)	0.130	(0.016)	0.122	(0.013)						
δ	0.137	(0.024)	0.192	(0.022)	0.168	(0.031)	0.194	(0.000)	0.239	(0.026)	0.227	(0.045)												
c			0.871	(0.122)	1.334	(0.362)			0.713	(0.095)	1.314	(1.305)			0.466	(0.063)	0.530	(0.113)			0.962	(0.094)	0.736	(0.038)
w_1			-0.674	(0.140)	-1.332	(0.218)			-0.632	(0.218)	-1.256	(0.772)			-0.286	(0.252)	-0.805	(0.330)			-0.418	(0.506)	-0.783	(0.241)
w_2			2.026	(0.314)	3.000	(0.437)			1.938	(0.636)	2.911	(1.450)			0.722	(0.798)	1.466	(0.854)			1.117	(1.509)	1.308	(0.669)
w_3			-1.882	(0.498)	-0.468	(0.492)			-1.692	(0.681)	-0.408	(1.480)			-0.029	(1.165)	1.060	(1.104)			-0.373	(1.732)	1.662	(0.863)
w_4			0.370	(0.985)	-4.362	(0.980)			0.235	(0.544)	-4.445	(6.642)			-1.166	(1.343)	-5.151	(1.582)			-0.668	(1.844)	-5.982	(1.057)
w_5			0.738	(1.138)	5.727	(1.627)			0.605	(0.772)	5.771	(10.502)			1.635	(1.373)	6.368	(1.716)			0.440	(2.174)	6.999	(1.298)
w_6			0.564	(0.773)	-1.189	(1.090)			0.474	(1.171)	-1.575	(4.702)			0.173	(1.283)	-0.871	(1.125)			1.001	(1.863)	-2.058	(1.531)
w_7			-5.860	(1.009)	-6.600	(1.753)			-5.225	(1.311)	-5.827	(3.282)			-5.863	(1.389)	-8.453	(2.149)			-5.400	(1.665)	-6.957	(2.570)
w_8			8.342	(1.637)	8.539	(1.679)			7.426	(1.262)	7.997	(1.943)			8.688	(1.551)	10.000	(1.689)			7.829	(1.922)	9.785	(2.378)
w_9			-4.116	(1.457)	-3.104	(0.968)			-3.014	(1.217)	-2.804	(3.835)			-4.278	(1.481)	-3.269	(1.066)			-3.860	(1.928)	-3.972	(1.353)
w_{10}			-0.486	(1.081)	-1.983	(0.993)			-1.745	(0.998)	-2.405	(4.409)			-1.089	(1.386)	-2.301	(1.086)			-1.057	(1.910)	-1.900	(0.982)
w_{11}			2.021	(0.868)	3.062	(1.029)			3.212	(1.035)	3.600	(3.912)			2.575	(1.351)	3.018	(1.252)			2.359	(2.149)	3.009	(1.007)
w_{12}			-1.660	(0.744)	-2.623	(0.880)			-2.686	(1.083)	-2.871	(2.801)			-1.462	(1.340)	-1.785	(1.251)			-1.136	(2.138)	-1.866	(1.072)
w_{13}			1.716	(0.739)	4.839	(1.106)			3.417	(0.950)	4.603	(3.546)			2.481	(1.643)	3.933	(1.325)			1.975	(2.456)	4.095	(1.268)
w_{14}			-1.846	(0.908)	-7.824	(1.689)			-5.281	(0.866)	-7.622	(4.738)			-5.155	(1.943)	-7.644	(1.583)			-4.909	(2.715)	-8.343	(1.480)
w_{15}			0.286	(0.999)	8.409	(2.383)			4.938	(1.715)	8.602	(6.186)			4.835	(2.432)	8.482	(2.208)			5.256	(3.321)	10.000	(1.696)
w_{16}																								
w_{17}																								
$Inflation$					-0.191	(0.145)					-0.281	(0.138)					-0.263	(0.125)					-0.395	(0.105)
$Inflation_V$					-10.000	(2.537)					-9.358	(19.123)					-9.413	(3.119)					-10.000	(1.798)
$InterestR$					0.163	(0.140)					0.119	(1.013)					0.332	(0.140)					0.191	(0.091)
$InterestR_V$					-2.888	(1.116)					-2.279	(6.446)					-0.806	(1.012)					-6.055	(1.211)
$unemp_V$					1.604	(1.196)					0.853	(2.361)					1.197	(1.172)					5.960	(4.520)
$USDCAD_V$					0.412	(0.143)					0.417	(0.148)					0.501	(0.137)					0.578	(0.107)
GDP					-0.036	(0.054)					-0.036	(0.063)					-0.023	(0.040)					-0.032	(0.034)
GDP_V					0.057	(0.059)					0.048	(0.240)					0.028	(0.044)					-0.022	(0.060)
$Persistence$	0.991		0.974		0.949		0.999		0.974		0.955		0.978		0.938		0.922		0.987		0.989		0.915	
BIC	2.306		2.313		2.326		2.312		2.324		2.337		2.312		2.321		2.332		2.327		2.373		2.354	
AIC	2.296		2.276		2.275		2.303		2.289		2.287		2.303		2.288		2.284		2.320		2.340		2.307	

Notes: This table presents the results of all volatility models for TSX. TSX data are for the period between 03/17/2003-03/31/2017. The sample size is 3500 observations. Only results with positivity constraints are reported. In addition to volatility models presented in the table we estimated EWMA model that resulted in smoothing parameter estimate and standard error given in brackets: $\lambda = 0.9369\ (0.0055)$ and information criteria: $AIC = 2.8222, BIC = 2.8240$.

Table 5: **Degree of Risk Aversion: SPX and TSX**

		GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA
SPX	Spline+Macro	-0.726	-0.57	-0.658	-0.158	
	Spline	-0.764	-0.6	-0.659	-0.183	
	No Spline	-0.755	-0.544	-0.659	-0.192	-0.146
TSX	Spline+Macro	-0.744	-0.614	-0.628	-0.213	
	Spline	-0.762	-0.638	-0.661	-0.245	
	No Spline	-0.715	-0.584	-0.632	-0.255	-0.19

Notes: This table presents correlation between returns r_t and log difference of fitted conditional variance $log(\sigma_t^2/\sigma_{t-1}^2)$ for each model. The more negative correlation implies higher degree of risk aversion in the model.

Table 6: **Forecasts of Volatility and Tail Risk: Low Volatility**

		GTARCH			GTARCH0			GJR-GARCH			GARCH			EWMA
	Forecast	NoSpline	Spline	SMacro	NoSpline	Spline	SMacro	NoSpline	Spline	SMacro	NoSpline	Spline	SMacro	
Panel A:	SPX	t = December 30, 2016												
	t+1day	10.875	9.656	10.841	10.762	8.399	9.409	10.178	9.328	10.383	9.615	8.481	9.116	8.044
σ	t+2day	11.065	9.717	10.949	10.976	8.437	9.468	10.345	9.391	10.516	9.814	8.595	9.256	
	t+3day	11.249	9.775	11.051	11.183	8.473	9.524	10.506	9.450	10.638	10.004	8.699	9.382	
VaR_{1day}	$q = 90\%$	0.854	0.772	0.867	0.849	0.683	0.771	0.820	0.765	0.858	0.787	0.713	0.759	0.647
	$q = 95\%$	1.199	1.065	1.185	1.179	0.930	1.033	1.113	1.032	1.138	1.073	0.947	0.995	0.892
	$q = 99\%$	1.842	1.647	1.822	1.841	1.430	1.601	1.735	1.550	1.741	1.655	1.436	1.540	1.414
VaR_{2day}	$q = 90\%$	1.208	1.092	1.226	1.201	0.966	1.090	1.160	1.082	1.214	1.113	1.008	1.074	
	$q = 95\%$	1.696	1.506	1.675	1.668	1.315	1.461	1.574	1.459	1.610	1.518	1.339	1.407	
	$q = 99\%$	2.605	2.330	2.576	2.604	2.022	2.264	2.454	2.192	2.462	2.341	2.030	2.178	
VaR_{3day}	$q = 90\%$	1.480	1.338	1.501	1.471	1.183	1.335	1.421	1.325	1.486	1.363	1.235	1.315	
	$q = 95\%$	2.077	1.844	2.052	2.043	1.611	1.789	1.928	1.787	1.971	1.859	1.640	1.723	
	$q = 99\%$	3.190	2.853	3.155	3.189	2.476	2.773	3.006	2.685	3.015	2.867	2.487	2.667	
ES_{1day}	$q = 90\%$	0.949	0.858	0.963	0.944	0.759	0.856	0.911	0.850	0.953	0.875	0.792	0.844	0.719
	$q = 95\%$	1.262	1.121	1.247	1.241	0.979	1.087	1.171	1.086	1.198	1.130	0.997	1.047	0.939
	$q = 99\%$	1.861	1.664	1.840	1.860	1.444	1.617	1.753	1.566	1.758	1.672	1.450	1.556	1.429
ES_{2day}	$q = 90\%$	1.342	1.214	1.362	1.335	1.073	1.211	1.289	1.202	1.348	1.237	1.120	1.193	
	$q = 95\%$	1.785	1.585	1.764	1.756	1.384	1.538	1.657	1.536	1.694	1.597	1.410	1.481	
	$q = 99\%$	2.631	2.353	2.602	2.630	2.042	2.287	2.479	2.215	2.487	2.364	2.051	2.200	
ES_{3day}	$q = 90\%$	1.644	1.486	1.668	1.635	1.314	1.483	1.579	1.472	1.651	1.515	1.372	1.461	
	$q = 95\%$	2.186	1.941	2.160	2.150	1.696	1.883	2.029	1.881	2.075	1.956	1.727	1.814	
	$q = 99\%$	3.223	2.882	3.187	3.222	2.501	2.801	3.036	2.712	3.046	2.896	2.512	2.694	
Panel B:	TSX	t = March 31, 2017												
	t+1day	4.273	3.711	4.111	4.132	3.679	4.018	4.262	3.659	4.108	4.011	3.625	4.089	3.627
σ	t+2day	4.325	3.738	4.149	4.191	3.724	4.073	4.290	3.666	4.133	4.052	3.653	4.142	
	t+3day	4.375	3.764	4.184	4.250	3.767	4.126	4.317	3.672	4.157	4.093	3.680	4.190	
VaR_{1day}	$q = 90\%$	0.346	0.307	0.344	0.339	0.307	0.335	0.356	0.308	0.336	0.340	0.310	0.351	0.299
	$q = 95\%$	0.472	0.413	1.185	0.456	0.414	0.454	0.478	0.413	0.461	0.459	0.416	0.474	0.410
	$q = 99\%$	0.724	0.611	1.822	0.722	0.625	0.685	0.743	0.625	0.681	0.728	0.636	0.720	0.646
VaR_{2day}	$q = 90\%$	0.490	0.434	1.226	0.479	0.434	0.473	0.503	0.436	0.475	0.481	0.438	0.496	
	$q = 95\%$	0.668	0.583	1.675	0.644	0.585	0.642	0.677	0.584	0.653	0.648	0.588	0.670	
	$q = 99\%$	1.024	0.865	2.576	1.021	0.883	0.968	1.050	0.884	0.963	1.029	0.900	1.018	
VaR_{3day}	$q = 90\%$	0.600	0.531	1.501	0.587	0.532	0.580	0.616	0.533	0.582	0.589	0.536	0.608	
	$q = 95\%$	0.818	0.715	2.052	0.789	0.717	0.787	0.829	0.715	0.799	0.794	0.720	0.821	
	$q = 99\%$	1.254	1.059	3.155	1.251	1.082	1.186	1.286	1.083	1.180	1.261	1.102	1.247	
ES_{1day}	$q = 90\%$	0.385	0.341	0.382	0.377	0.341	0.372	0.395	0.342	0.379	0.378	0.344	0.390	0.332
	$q = 95\%$	0.497	0.434	0.481	0.480	0.436	0.478	0.504	0.435	0.493	0.483	0.438	0.499	0.432
	$q = 99\%$	0.731	0.618	0.688	0.729	0.631	0.692	0.750	0.632	0.699	0.735	0.643	0.727	0.653
ES_{2day}	$q = 90\%$	0.544	0.482	0.541	0.533	0.482	0.526	0.559	0.484	0.535	0.535	0.487	0.552	
	$q = 95\%$	0.703	0.614	0.680	0.678	0.616	0.676	0.712	0.615	0.697	0.683	0.619	0.705	
	$q = 99\%$	1.034	0.873	0.973	1.032	0.892	0.978	1.061	0.893	0.988	1.040	0.909	1.028	
ES_{3day}	$q = 90\%$	0.667	0.590	0.662	0.652	0.591	0.644	0.685	0.593	0.656	0.655	0.596	0.676	
	$q = 95\%$	0.861	0.752	0.832	0.831	0.755	0.828	0.872	0.753	0.854	0.836	0.758	0.864	
	$q = 99\%$	1.267	1.070	1.192	1.263	1.093	1.198	1.299	1.094	1.210	1.273	1.114	1.259	

Notes: This table presents 1 to 3 day forecasts of volatility, VaR and ES produced by each volatility model for SPX and TSX at the time of low volatility. VaR and ES were estimated using Hull and White (1998) method.

Table 7: **Forecasts of Volatility and Tail Risk: High Volatility**

		GTARCH			GTARCH0			GJR-GARCH			GARCH			EWMA
	Forecast	NoSpline	Spline	SMacro	NoSpline	Spline	SMacro	NoSpline	Spline	SMacro	NoSpline	Spline	SMacro	
Panel A:	SPX	t = November 11, 2008												
	t+1day	91.214	88.806	84.678	83.340	79.724	79.610	81.014	80.449	77.198	66.241	64.261	62.656	71.063
σ	t+2day	90.654	87.345	83.405	82.996	78.529	78.561	80.039	78.526	75.390	65.520	63.063	61.794	
	t+3day	90.098	85.924	82.168	82.654	77.374	77.553	79.077	76.685	73.670	64.808	61.927	60.994	
VaR_{1day}	$q = 90\%$	7.385	7.130	6.771	6.787	6.628	6.562	6.611	6.669	6.412	5.560	5.451	5.285	5.717
	$q = 95\%$	10.119	9.726	9.044	9.138	8.683	8.475	8.893	8.845	8.113	7.490	7.205	6.581	7.881
	$q = 99\%$	14.558	14.461	13.050	13.962	12.585	12.886	12.726	12.163	11.703	10.608	9.938	9.714	12.494
VaR_{2day}	$q = 90\%$	10.444	10.083	9.575	9.599	9.373	9.280	9.350	9.431	9.068	7.863	7.709	7.475	
	$q = 95\%$	14.311	13.755	12.791	12.923	12.280	11.986	12.577	12.508	11.473	10.593	10.189	9.307	
	$q = 99\%$	20.589	20.451	18.455	19.746	17.799	18.223	17.997	17.202	16.550	15.001	14.054	13.738	
VaR_{3day}	$q = 90\%$	12.792	12.349	11.727	11.756	11.480	11.365	11.451	11.551	11.106	9.630	9.441	9.155	
	$q = 95\%$	17.527	16.846	15.665	15.828	15.040	14.679	15.404	15.320	14.052	12.973	12.479	11.399	
	$q = 99\%$	25.216	25.048	22.603	24.183	21.799	22.319	22.042	21.068	20.270	18.373	17.213	16.825	
ES_{1day}	$q = 90\%$	8.206	7.922	7.523	7.541	7.364	7.291	7.346	7.410	7.124	6.177	6.057	5.873	6.352
	$q = 95\%$	10.652	10.238	9.520	9.619	9.140	8.921	9.361	9.310	8.540	7.884	7.584	6.928	8.296
	$q = 99\%$	14.705	14.607	13.181	14.103	12.713	13.016	12.855	12.286	11.821	10.715	10.038	9.812	12.620
ES_{2day}	$q = 90\%$	11.605	11.203	10.639	10.665	10.414	10.311	10.388	10.479	10.075	8.736	8.565	8.305	
	$q = 95\%$	15.064	14.479	13.464	13.604	12.926	12.616	13.239	13.167	12.077	11.150	10.725	9.797	
	$q = 99\%$	20.797	20.658	18.641	19.945	17.978	18.407	18.179	17.375	16.717	15.153	14.196	13.877	
ES_{3day}	$q = 90\%$	14.213	13.721	13.030	13.062	12.755	12.628	12.723	12.834	12.340	10.700	10.490	10.172	
	$q = 95\%$	18.449	17.733	16.490	16.661	15.832	15.452	16.214	16.126	14.792	13.656	13.136	11.999	
	$q = 99\%$	25.471	25.301	22.831	24.428	22.019	22.544	22.265	21.280	20.474	18.558	17.387	16.995	
Panel B:	TSX	t = September 16, 2008												
	t+1day	14.694	17.452	16.305	13.449	16.265	15.233	15.724	17.760	17.229	14.554	16.287	15.911	13.363
σ	t+2day	14.649	17.404	16.149	13.459	16.295	15.183	15.570	17.541	16.913	14.477	16.218	15.760	
	t+3day	14.604	17.358	15.999	13.469	16.325	15.135	15.417	17.333	16.617	14.400	16.152	15.621	
VaR_{1day}	$q = 90\%$	1.183	1.366	1.322	1.091	1.319	1.236	1.279	1.415	1.388	1.210	1.330	1.313	1.101
	$q = 95\%$	1.648	1.862	1.185	1.533	1.749	1.657	1.771	1.936	1.880	1.660	1.827	1.785	1.511
	$q = 99\%$	2.573	2.794	1.822	2.377	2.689	2.486	2.810	2.834	2.856	2.648	2.625	2.706	2.382
VaR_{2day}	$q = 90\%$	1.674	1.933	1.226	1.543	1.865	1.748	1.808	2.002	1.963	1.711	1.880	1.857	
	$q = 95\%$	2.331	2.634	1.675	2.169	2.473	2.344	2.504	2.738	2.658	2.347	2.584	2.524	
	$q = 99\%$	3.639	3.951	2.576	3.361	3.803	3.515	3.974	4.008	4.039	3.745	3.713	3.827	
VaR_{3day}	$q = 90\%$	2.050	2.367	1.501	1.890	2.284	2.141	2.215	2.451	2.404	2.096	2.303	2.274	
	$q = 95\%$	2.855	3.225	2.052	2.656	3.029	2.870	3.067	3.354	3.256	2.875	3.165	3.091	
	$q = 99\%$	4.456	4.838	3.155	4.117	4.658	4.306	4.867	4.908	4.947	4.587	4.547	4.687	
ES_{1day}	$q = 90\%$	1.315	1.518	1.469	1.213	1.465	1.373	1.421	1.573	1.393	1.344	1.477	1.459	1.223
	$q = 95\%$	1.735	1.960	1.855	1.614	1.841	1.744	1.864	2.038	1.930	1.747	1.923	1.879	1.590
	$q = 99\%$	2.599	2.822	2.595	2.401	2.717	2.511	2.838	2.862	2.880	2.675	2.652	2.734	2.406
ES_{2day}	$q = 90\%$	1.859	2.147	2.077	1.715	2.072	1.942	2.009	2.224	1.970	1.901	2.089	2.063	
	$q = 95\%$	2.454	2.772	2.624	2.283	2.604	2.467	2.636	2.882	2.729	2.471	2.720	2.657	
	$q = 99\%$	3.675	3.991	3.670	3.395	3.842	3.551	4.014	4.048	4.073	3.783	3.750	3.866	
ES_{3day}	$q = 90\%$	2.277	2.630	2.544	2.100	2.538	2.379	2.461	2.724	2.412	2.329	2.559	2.527	
	$q = 95\%$	3.005	3.395	3.214	2.796	3.189	3.021	3.228	3.530	3.342	3.026	3.331	3.254	
	$q = 99\%$	4.501	4.887	4.495	4.158	4.705	4.349	4.916	4.958	4.988	4.633	4.593	4.735	

Notes: This table presents 1 to 3 day forecasts of volatility, VaR and ES produced by each volatility model for SPX and TSX at the time of high volatility. VaR and ES were estimated using Hull and White (1998) method.

Table 8: **Backtesting for SPX and TSX VaR Models**

Upper and lower bound from the Kupiec Test		
Breaches allowed at 95% CI	LB	UB
VaR_{Q90}	310	350
VaR_{Q95}	146	175
VaR_{Q99}	22	35

Breaches for SPX Data

		GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA
Spline	90% VaR	343	336	336	307	
	95% VaR	169	166	167	160	
	99% VaR	32	33	33	29	
Spline + Macro Variable	90% VaR	336	327	326	308	
	95% VaR	171	167	166	160	
	99% VaR	34	29	32	30	
No Spline	90% VaR	336	327	326	308	354
	95% VaR	171	167	166	160	176
	99% VaR	34	29	32	30	36

Breaches for TSX Data

		GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA
Spline	90% VaR	341	328	327	321	
	95% VaR	168	163	162	155	
	99% VaR	34	32	31	30	
Spline + Macro Variable	90% VaR	330	329	332	312	
	95% VaR	164	152	162	151	
	99% VaR	33	33	33	27	
No Spline	90% VaR	330	329	332	312	356
	95% VaR	164	152	162	151	182
	99% VaR	33	33	33	27	36

Notes: This table presents number of backtest breaches for VaR of SPX and TSX produced by each volatility model. VaR was estimated using Hull and White (1998) method.

Table 9: **Threshold Regimes: SPX and TSX**

	Model	Thresholds (in %)		Proportion			Unit Root		
				Low regime	Middle regime	High regime	Low regime	Middle regime	High regime
Panel B: SPX Data									
No Spline	GTARCH	1.90	2.44	33%	25%	42%	NO	NO	NO
	GTARCH0	1.91	2.95	30%	41%	28%	NO	NO	NO
	GJR-GARCH	1.88	2.39	31%	25%	44%	NO	NO	NO
	GARCH	1.93	2.47	27%	29%	44%	NO	NO	NO
	EWMA	1.82	2.90	25%	44%	31%	NO	NO	NO
	Average	1.89	2.63	29%	33%	38%			
Spline	GTARCH	1.74	2.66	26%	39%	35%	NO	NO	NO
	GTARCH0	1.92	2.47	33%	26%	41%	NO	NO	NO
	GJR-GARCH	1.87	2.47	36%	26%	38%	NO	NO	NO
	GARCH	1.86	2.29	27%	25%	48%	NO	NO	NO
	Average	1.85	2.48	31%	29%	40%			
Spline + Macro Variables	GTARCH	1.74	2.74	28%	40%	32%	NO	NO	NO
	GTARCH0	1.78	2.74	25%	41%	34%	NO	NO	NO
	GJR-GARCH	1.74	2.18	26%	25%	49%	NO	NO	NO
	GARCH	1.85	2.85	27%	43%	30%	NO	NO	NO
	Average	1.78	2.63	26%	37%	36%			
Overall	**Average**	1.84	2.58	29%	33%	38%			
Panel B: TSX Data									
No Spline	GTARCH	0.74	0.97	29%	29%	42%	NO	NO	NO
	GTARCH0	0.77	1.10	27%	41%	32%	NO	NO	NO
	GJR-GARCH	0.74	1.01	26%	33%	40%	NO	NO	NO
	GARCH	0.83	1.05	32%	29%	40%	NO	NO	NO
	EWMA	0.79	1.02	34%	26%	40%	NO	YES	YES
	Average	0.77	1.03	30%	32%	39%			
Spline	GTARCH	0.73	0.94	33%	26%	41%	NO	YES	YES
	GTARCH0	0.87	1.15	48%	25%	27%	NO	YES	YES
	GJR-GARCH	0.71	0.93	26%	30%	44%	NO	YES	YES
	GARCH	0.74	0.95	25%	27%	48%	NO	YES	YES
	Average	0.76	0.99	33%	27%	40%			
Spline + Macro Variables	GTARCH	0.82	1.12	47%	26%	27%	NO	YES	NO
	GTARCH0	0.82	1.04	41%	26%	33%	NO	YES	NO
	GJR-GARCH	0.70	0.87	25%	27%	47%	NO	NO	NO
	GARCH	0.75	1.05	26%	39%	36%	NO	YES	NO
	Average	0.77	1.02	35%	29%	36%			
Overall	**Average**	0.77	1.01	32%	29%	38%			

Notes: This table presents results of three regime threshold autoregressive (3TAR) model applied to VaR produced by each volatility model for SPX and TSX. VaR was estimated using Hull and White (1998) method.

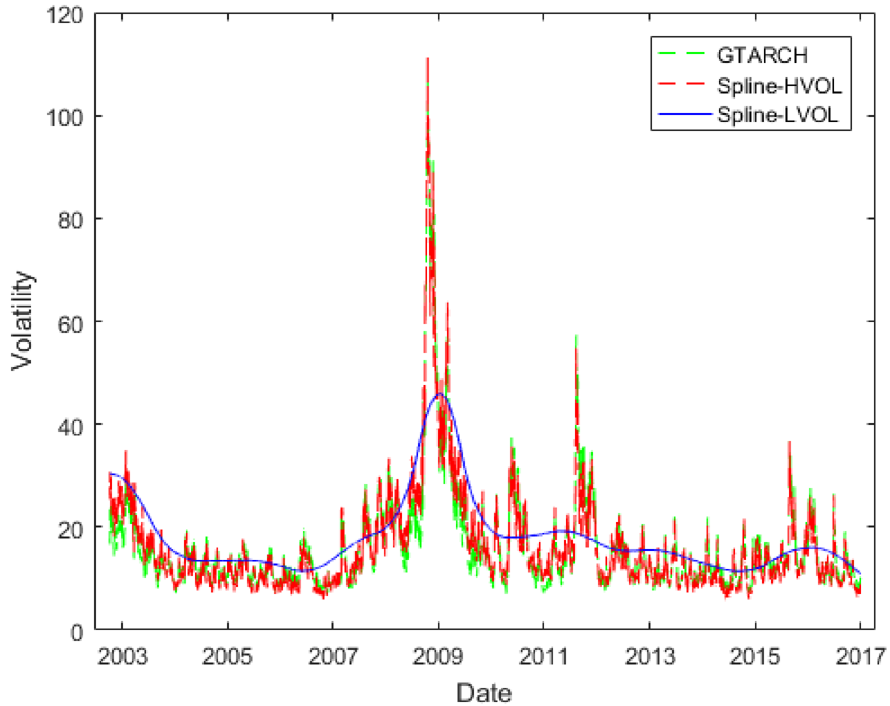

Figure 1: **High and Low Frequency Volatility: Spline-GTARCH for SPX**

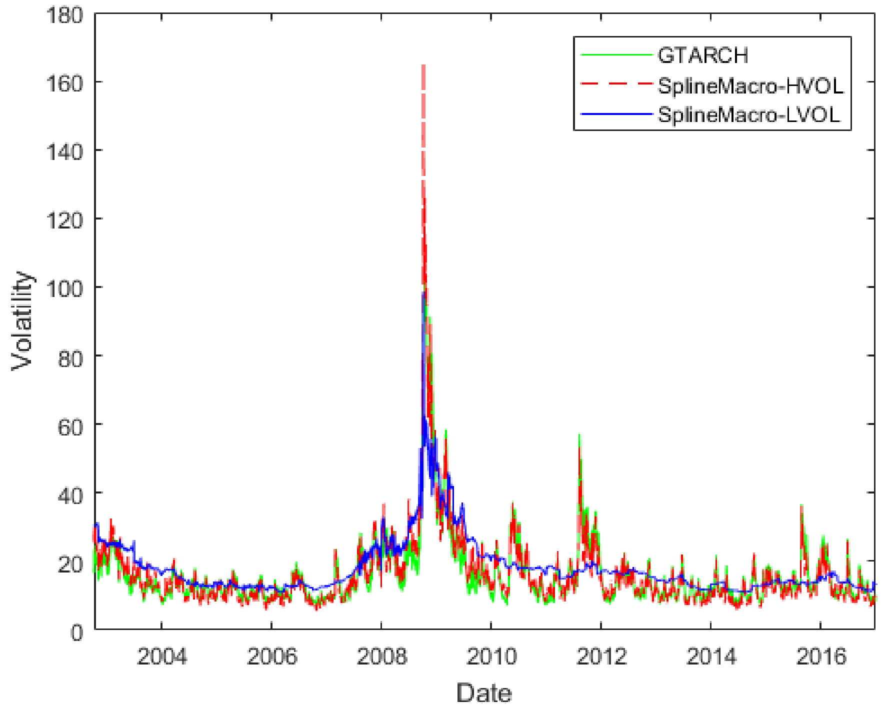

Figure 2: **High and Low Frequency Volatility: Spline-Macro-GTARCH for SPX**

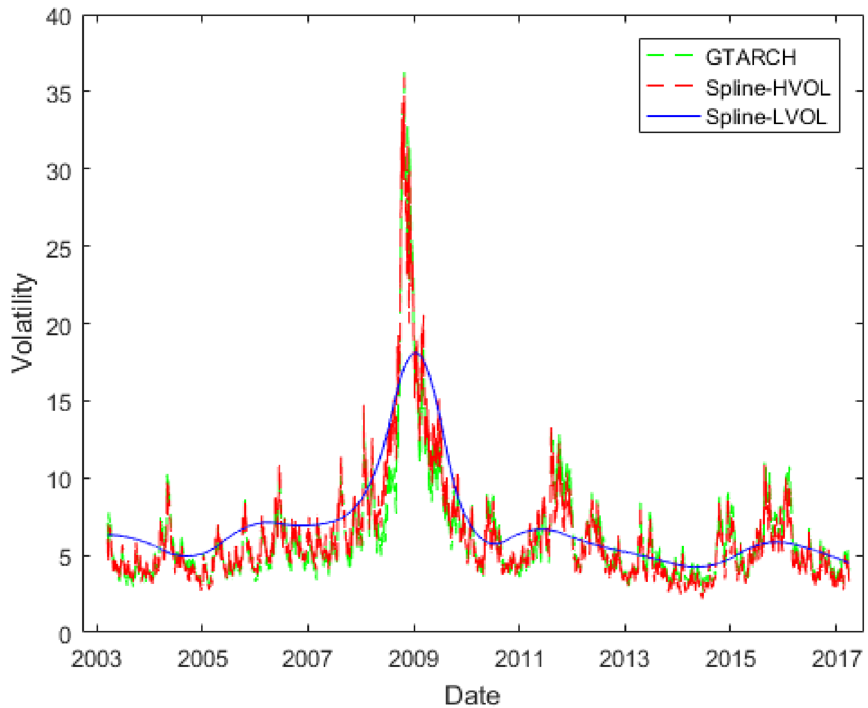

Figure 3: **High and Low Frequency Volatility: Spline-GTARCH for TSX**

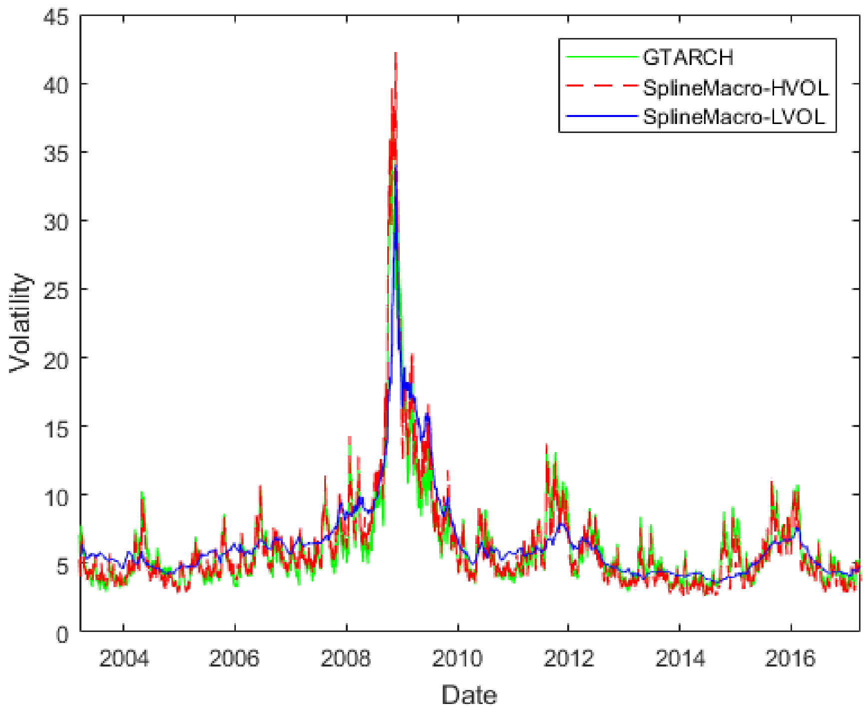

Figure 4: **High and Low Frequency Volatility: Spline-Macro-GTARCH for TSX**

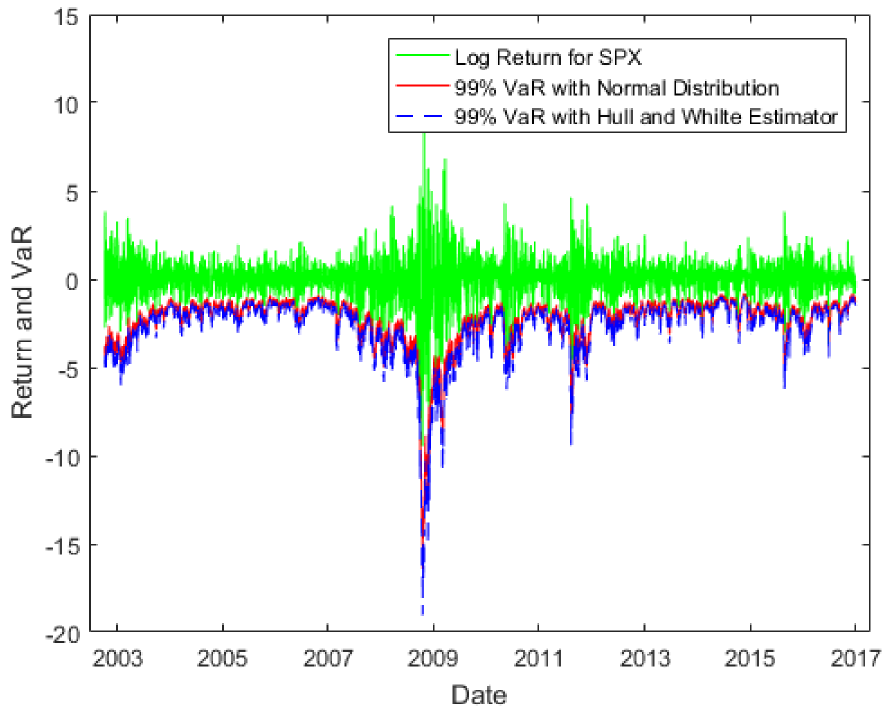

Figure 5: **SPX Log Returns and 1 day VaR: Spline-GTARCH**

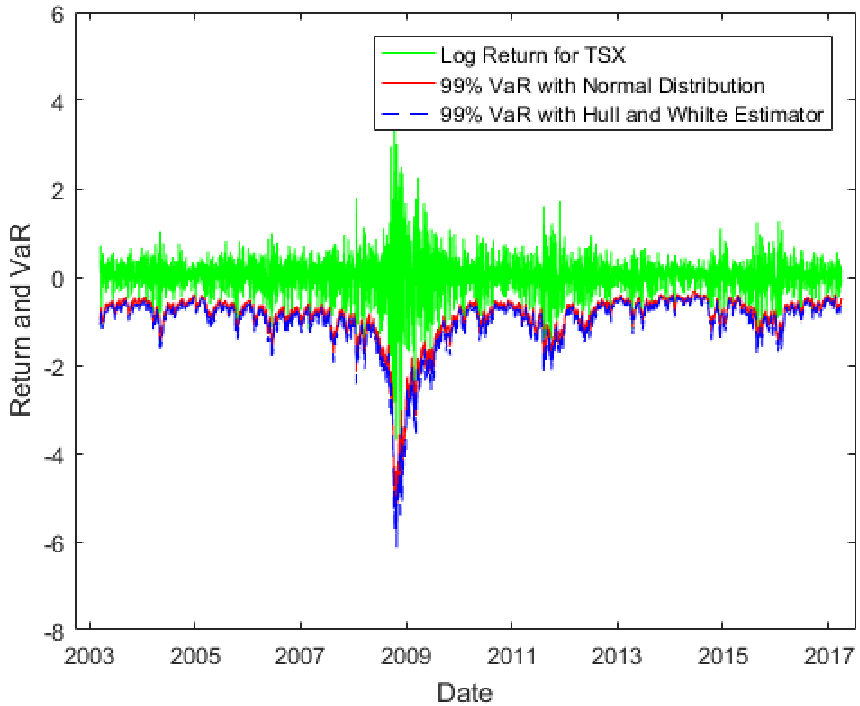

Figure 6: **TSX Log Returns and 1 day VaR: Spline-GTARCH**

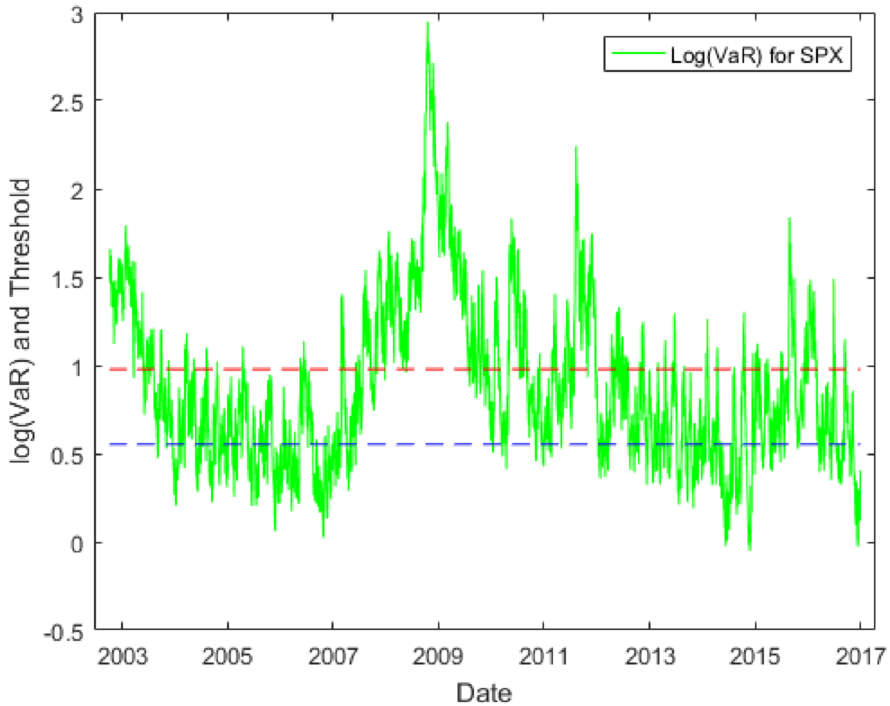

Figure 7: **Estimated Thresholds with 3 Regimes: Log(VaR) Spline-GTARCH for SPX**

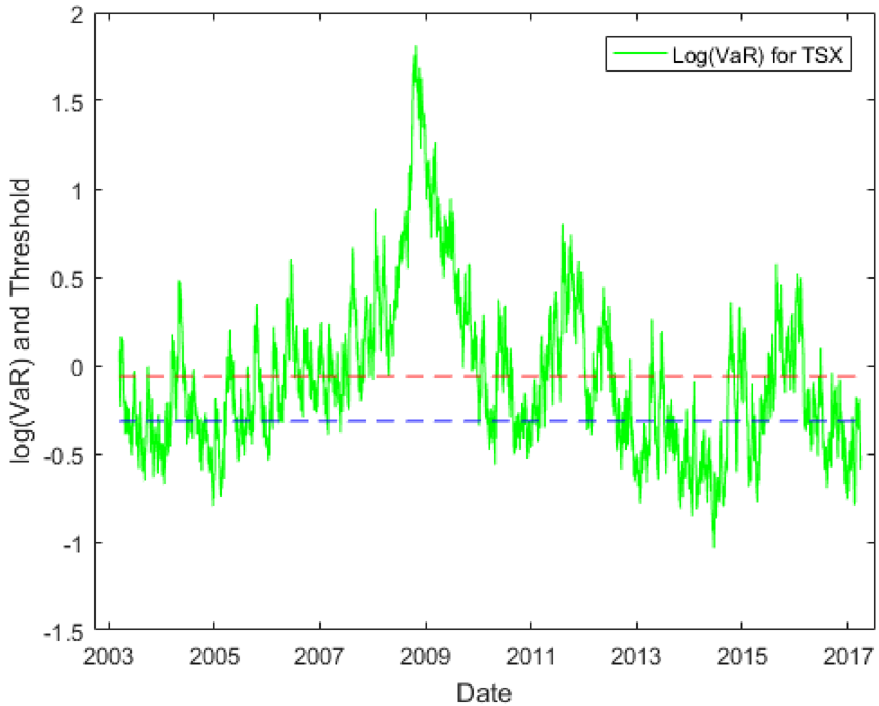

Figure 8: **Estimated Thresholds with 3 Regimes: Log(VaR) Spline-GTARCH for TSX**

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